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Unit Plan

EDC 292B

Ms. Slaine

Content Area: College Algebra

Grade Level: High School Course

**Unifying Concepts: Quadratics**

**Grade Level Expectations**

1.Quantitative reasoning is used to make sense of quantities and their relationships in problem situations MA10-GR.HS-S.1-GLE.2

2. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables MA10-GR.HS-S.2-GLE.1

3. Quantitative relationships in the real world can be modeled and solved using functions MA10-GR.HS-S.2-GLE.2

3. Expressions can be represented in multiple, equivalent forms MA10-GR.HS-S.2-GLE.3

4. Solutions to equations, inequalities and systems of equations are found using a variety of tools MA10-GR.HS-S.2-GLE.4

**21st Century Skills Critical Thinking and Reasoning**

Thinking Deeply, Thinking Differently, Information Literacy

**Mathematical Practices**

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

**Unit Title: Quadratics**

Focus: Maximization and Minimization Problems in Physics, Economics, and Business.

**Inquiry Questions**

What is the maximum height an object in flight reaches?

What is the maximum profit a business can realize?

When does the cost function reach a minimum?

**Unit Strands**

Functions: Equation of Quadratics; Graphing Quadratics: Vertex, Shape

Algebra: Completing the Square, Quadratic Formula, Factoring

**Generalizations**

My students will understand that completing the square, factoring and using the quadratic formula provide a mechanism or transforming and solving quadratic equations.

My students will understand that mathematicians derive quadratic functions to model parabolic behavior.

My students will understand that transformation of quadratic functions allows for solution of many practical problems.

**Questions**

1. How do complete the square?
2. How do we factor equations?
3. How do we use the quadratic formula?
4. How do we find the vertex and shape of a parabola?
5. Why do we study quadratic equations?
6. What is a model?
7. How do we model real world phenomena using quadratic equations?

**Key Knowledge and Skills**

1. Completing the square requires the skills of addition, division and squaring as well as knowledge of PEMDAS.
2. Factoring requires the skill of identifying the sum and product of an expression.
3. The quadratic formula may be used if completing the square and factoring are not effective means for solution.
4. The vertex and shape of a parabola can be determined by looking at the standard form of a quadratic equation.
5. Modeling real world phenomena involves taking word problems and translating those problems into mathematics and then interpreting those problems again using English.

**Vocabulary**

**Academic Vocabulary:** Graph, interpret, analyze, evaluate, solve, simplify.

**Technical Vocabulary:** X-intercept, Y-intercept, Vertex, completing the square, maximum, minimum.

**Unit Description**

This unit focuses on quadratic functions that are part of our everyday lives. Students explore various types of maximization and minimization problems from different areas of study. As they explore these different types of problems, they are motivated to solve quadratic equations, necessitating the introduction of different problem solving techniques. The students become fluent in rewriting expressions and solving for unknown variables in quadratic equations. In addition, students explore cost and revenue, height and distance, area and perimeter.

**Performance Assessment:** The summative assessment for this unit.

1. The Profit P that a company makes depends on the amount x (in hundreds of dollars) the company spends on advertising according to the model P = $-\frac{1}{2}x^{2} $+ 20x + 230. What expenditure for advertising will yield a maximum profit?

2) A manufacturer of lighting fixtures has daily production costs of

C = $\frac{1}{4}x^{2} $-10x + 800

Where C is the total cost (in dollars) and x is the number of units produced. How many fixtures should be produced each day to yield a minimum cost? What is the cost?

3) The height of a football in the air t seconds after it is thrown can be modeled by the function

Y = -16t2 + 96t + 3, where y are measured in feet and t is time. What is the maximum height that the football reaches? How long did it take the football to reach the maximum height?

At what times will the football be above the ground at 122 feet?

4) $f\left(x\right)=-2\left(x+3\right)^{2}-5$

a. Identify the vertex

b. Identify the y-intercept

c. Identify the x intercept

5) Consider $h\left(x\right)=2x^{2}+12x+10$. Identify the vertex by completing the square, x-intercepts, and the y-intercept

6) Find the vertex by completing the square.

 $x^{2}-2x-24=0$

 **Differentiation: Multiple Modes for Student Expression**

1. Students can express their interpretations by verbally explaining the steps they used to complete quadratic word problems and functional problems involving completing the square, factoring and the quadratic formula.
2. Students can work in collaborative groups on quadratic problems building on each other’s strengths to solve problems and can present those problems using either a document camera or a whiteboard.
3. Students can use a graphing calculator to arrive at solutions and graph solutions.
4. Students can extend quadratic analysis to solve maximization and minimization problems in a written format.
5. Students can interpret mathematical solutions using everyday English vocabulary and can write down their interpretations.

**Ongoing Specific Learning Experiences**

1. Students can express quadratic functions and solutions and can critique the reasoning and steps of others by providing a justification for a solution through a series of logical steps using correct terminology.
2. Students can engage in the practice of mathematical modeling to express the solution to real world problems by mapping relationships between quadratic functions and interpret results with regard to whether they make sense or not.
3. Students are fluent with translating between different types of quadratic functions based on the parametric values of the standard form.

**Anticipatory Set/Prior Knowledge and Experiences**

Students have studied quadratic functions I Algebra II and know how to graph quadratic functions and make a chart of points on the coordinate plane. This ability to create a table of values and then graph a function will serve as a building block for manipulating and solving a wide variety of types of quadratic equations and the interpreting the solutions of those equations.

**Learning Experience #1**

I will introduce a scenario to students where a helicopter is dropping bales of hay onto the ground to feed animals. The bales of hay are dropped from a specified amount of feet at a specified velocity which can be described using a quadratic equation so that students can determine the maximum height the bale reaches and the time it takes to hit the ground.

Students can estimate how long it will take the bale to hit the ground based on different heights and velocities of the bale of hay and can then create a table and graph in their graphing calculator that shows the path of the bale of hay. Students will also be able to show how to find the vertex of the bale of hay using the quadratic equation they are given to determine how high the bale of hay goes.

Assessment: Students mastering the concept and skills of this lesson will be able to answer questions such as:

Why is it possible to estimate the height and time it takes for the bale of hay to hit the ground?

What does the formula about the flight of the bale of hay show about the nature of parabolic flight based on initial conditions?

Differentiation: Teacher provides the steps to derive answers using Algebra; Teacher models how to use the graphing calculator; Teacher models how to create a chart of values to plot so that students can solve problems using algebraic manipulation and a graphing calculator.

Extensions: Students can view videos on the projected flight of objects.

**Learning Experience #2**

I will set up a situation where a homeowner wishes to enclose a small rectangular area with a fence for use as a garden or yard or play area. The goal of the homeowner is to make the area of the rectangular area as large as possible for the garden or view or child. Students can imagine how a homeowner might experiment with different lengths and widths to find the area that is the largest.

Students can then estimate what the dimensions need to be to determine the maximum area. They can make a chart showing length, width and area and can using a graphing calculator to determine the shape of the area function and the vertex and different sizes of yard.

Assessment: Students mastering the concept and skills of this lesson will be able to answer questions such as:

When does the area function reach a maximum based on the dimensions?

What are the reasons for having a yard as large as possible?

Differentiation: Teacher provides the steps to derive the maximum area using Algebra; Teacher models how to use the graphing calculator to plot the area as a function of length and width; Teacher models how to create a chart of values to plot so that students can solve problems using algebraic manipulation and a graphing calculator.

Extensions: Students can view videos on space usage by businesses and the factors that go into purchasing space.

**Learning Experience #3**

I will begin with an investment problem where a person with a specified amount of cash makes an initial investment is compounded yearly for a period of 2 years and then determine the amount that the initial investment grows to after the 2 year period is up and the total amount the person will have after two years so that students can determine the total amount of money a person will have based on compounding period, time and interest.

Students can estimate how long it will take the investment to grow to a specified amount and can estimate how compounding period and interest affect the growth rate of the investment.

Assessment: Students mastering the concept and skills of this lesson will be able to answer questions such as:

What is it possible to estimate the height and time it takes for the bale of hay to hit the ground?

What does the formula about the flight of the bale of hay show about the nature of parabolic flight based on initial conditions?

Differentiation: Teacher provides the steps to derive answers using Algebra; Teacher models how to use the graphing calculator; Teacher models how to create a chart of values to plot so that students can solve problems using algebraic manipulation and a graphing calculator.

Extensions: Students can view videos on the projected flight of objects.

Learning Experience #4

I will start this experience with a business scenario where a company seeks to maximize profits as a function of advertising with a specific profit function in which costs are modeled quadratically and revenues are modeled as a linear function. Spending in hundreds of dollars can be associated with different integer values ranging from 1 to 10 and profits will at first increase with spending and then decline as more and more spending takes place.

Students can estimate intervals for which profits are positive and increasing, positive and decreasing and then what amount of spending will produce 0 profits do to oversaturation.

Assessment: Students mastering the concept and skills of this lesson will be able to answer questions such as:

What expenditure will yield maximimum profit?

How is it possible to estimate the profit which is highest?

Differentiation: Teacher provides the steps to derive answers using Algebra; Teacher models how to use the graphing calculator to estimate maximum profits and identify profit levels based on spending level.

Extensions: Students can view business videos on profit maximimization.

References

### Baumann, James. 2012-2013. College Algebra Lectures and Problems. Unpublished.

### Colorado Department of Education. *Teacher Instructional Unit Samples. Algebra II Unit Title: Logarithmic Log Jams (Lake County School District)*Retrieved from: https://www.cde.state.co.us/standardsandinstruction/instructionalunits-math .